This behavior was observed also for higher values of β. Furthermore, when consideration is given to the fact that the radiative-conductive interaction is most significant for an optical depth of order 1, such a behavior, then, can be further justified.

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Solidification Heat Transfer on a Moving Continuous Cylinder

F. B. Cheung* and S. W. Cha†
Pennsylvania State University,
University Park, Pennsylvania

Introduction

WHEN a continuous cylinder is caused to travel axially through a large bath of warm liquid, a boundary-layer type of flow will be induced in the liquid phase adjacent to the solid boundary.[1] If the cylinder is precooled below the freezing point of the liquid before entering the bath, a solidified layer or freeze coat will form on the surface of the moving object. This solidification process, known as freeze-coating, is an important material manufacturing process with applications in the electrical and chemical industries for casting insulating coatings on metal wires or electricity cables.

Seeniraj and Bose[2] studied the freeze-coating problem by assuming the moving object to be isothermal and the liquid to be saturated at its freezing point. Cheung[3,4] investigated the problem of freeze-coating of a superheated liquid on a continuous moving plate by including the effects of temperature variation within the plate and heat convection from the warm liquid. For a moving cylinder, two asymptotic solutions were presented by Cheung and Cha.[5,6] The first solution was obtained under the condition of negligible interaction between the freeze coat and the liquid flowfield; the second solution was obtained under the condition of negligible radial variation of the cylinder temperature.

In this study, a general solution is sought by the finite-difference method for freeze-coating of a superheated liquid on a nonisothermal axially moving cylinder, taking full account of mutual interaction between the freeze coat and the liquid flowfield, as well as variation of the cylinder temperature in the radial and axial directions.

Mathematical Model

To formulate the problem mathematically, the frame of axes are chosen to be stationary with respect to the liquid bath (Fig. 1). The freeze-coating process is considered to take place at a steady state and at a constant freezing temperature of the liquid. Under these conditions, the equations governing the liquid velocity, liquid temperature, freeze-coat temperature, cylinder temperature, and the axial variation of the freeze-coat thickness can be written as follows:

Boundary-Layer Flow Region, $r \geq \delta(x)$

$$r\frac{\partial u}{\partial x} + \frac{\partial (vr)}{\partial r} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{3}$$

$$x = 0: \quad u = 0, \qquad T = T_\infty > T_f \tag{4a}$$

$$r = \delta(x): \quad u = U, \qquad v = 0, \quad T = T_f \tag{4b}$$

$$r \to \infty: \quad u = 0, \qquad T = T_\infty \tag{4c}$$

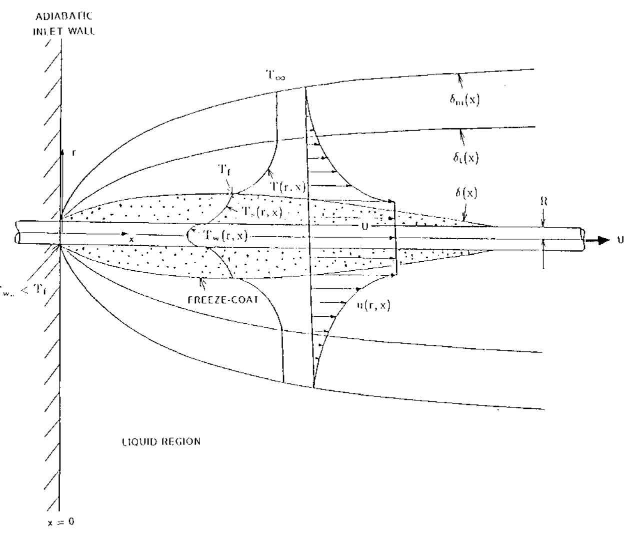

Fig. 1 Schematic of the freeze-coat on a nonisothermal axially moving continuous cylinder, indicating nomenclature.

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*Associate Professor, Mechanical Engineering Department.

†Graduate Assistant, Mechanical Engineering Department; currently, Assistant Professor, Suwon University, South Korea.

Freeze-Coat Region, $R \le r \le \delta(x)$

$$\rho_c C_{p_s} U \frac{\partial T_s}{\partial x} = \frac{k_s}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_s}{\partial r}\right) \tag{5}$$

$$x = 0: \quad \delta(x) = R \tag{6a}$$

$$r = R: \quad T_s = T_w, \qquad k_s \frac{\partial T_s}{\partial r} = k_w \frac{\partial T_w}{\partial r} \tag{6b}$$

$$r = \delta(x): \quad T_s = T_f, \qquad \rho_s U_c \lambda \frac{\mathrm{d}\delta}{\mathrm{d}x} = k_s \frac{\partial T_s}{\partial r} - k\frac{\partial T}{\partial r} \tag{6c}$$

Wall Region $0 \le r \le R$

$$\rho_w C_{p_w} U \frac{\partial T_w}{\partial x} = \frac{k_w}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_w}{\partial r}\right) \tag{7}$$

$$x = 0: \quad T_w = T_{w_0} < T_f \tag{8a}$$

$$r = 0: \quad \frac{\partial T_w}{\partial r} = 0 \tag{8b}$$

$$r = R: \quad T_w = T_s, \qquad k_w \frac{\partial T_w}{\partial r} = k_s \frac{\partial T_s}{\partial r} \tag{8c}$$

The local coefficient h_x of convective heat transfer from the liquid to the freeze-coat can be calculated once the distribution of the liquid temperature T in the boundary-layer flow region is known. This is given by

$$Nu_x = \frac{h_x R}{k} = \frac{R(\partial T/\partial r)|r = \delta(x)}{T_\infty - T_f} \tag{9}$$

In the preceeding formulation, u and v are the axial and radial velocity components, respectively, T is the temperature, ρ density, C_p specific heat, k thermal conductivity, α thermal diffusivity, and λ the latent heat of fusion. The subscript s refers to the freeze-coat, w the cylinder wall, and the nonsubscripted quantities refer to the liquid flowfield.

Numerical Solution

To facilitate the numerical computation process, the solution domain is transformed into a more tactical shape. This is done by immobilizing the solid-liquid interface and stretching the coordinate using Landau Transformation technique:

$$\xi = (U_c x/\alpha_s), \qquad \eta = (r/R) \quad \text{for the wall region} \tag{10a}$$

$$\eta = 1 + (r - R)/(\delta - R) \quad \text{for the freeze-coat region} \tag{10b}$$

$$\eta = 3 - \exp\left[\omega\left(1 - \frac{r}{\delta}\right)\right] \quad \text{for the boundary-layer flow region} \tag{10c}$$

In the transformed coordinates, the solid-liquid interface remains fixed at $\eta = 2$. The wall region is contained between $0 \le \eta \le 1$, the freeze-coat region between $1 \le \eta \le 2$, and the boundary-layer flow region between $2 \le \eta \le 3$. The stretching factor ω is employed in Eq. (10c) to expand the flow region near the interface. This expansion is needed to ensure that enough grid points are present there in the numerical computation. Meanwhile, the dependent variables are nondimensionalized according to

$$\Delta = \frac{\delta - R}{R}\ \theta = \frac{T - T_{w_o}}{T_f - T_{w_o}}, \qquad \theta_s = \frac{T_s - T_{w_o}}{T_f - T_{w_o}} \tag{11a}$$

$$\theta_w = \frac{T_w - T_{w_o}}{T_f - T_{w_o}}, \qquad U = \frac{u}{U_c}, \qquad V = \frac{v}{U_c} \tag{11b}$$

To avoid numerical instability, a fully implicit scheme is employed in the solution of the dimensionless governing equations.

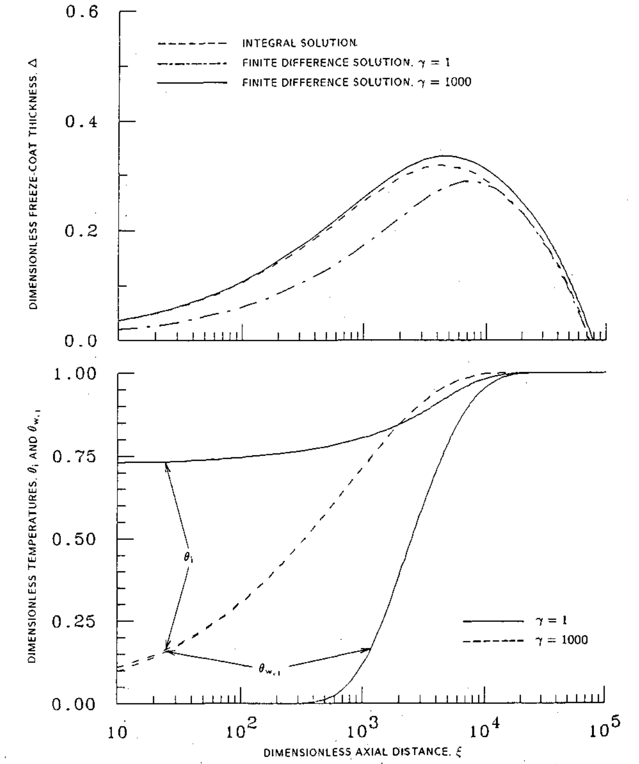

Fig. 2 Effect of the thermal conductivity ratio: a) comparison between the finite-difference and integral solutions; and b) axial variations of the dimensionless wall temperatures.

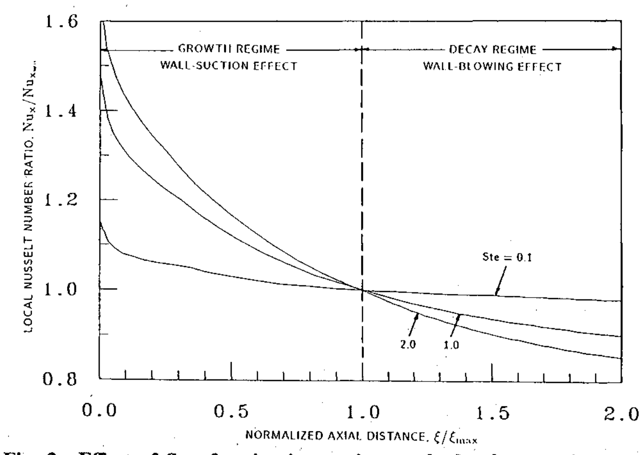

Fig. 3 Effect of flow-freezing interaction on the local convective heat transfer at the solid-liquid interface.

Results and Discussion

The present finite-difference solution is compared to the asymptotic integral solution[6] as shown in Fig. 2a. For the case of $\gamma = 1000$, where $\gamma = k_w/k_s$ is the thermal conductivity ratio, the integral solution[6] and the finite difference solution are almost identical. For the cases of $\gamma = 1$, however, the integral solution overestimates the freeze-coat thickness, although the total lifetime is almost the same as the one obtained by the finite difference solution, regardless of the value of γ. As shown in Fig. 2b, for the case of $\gamma = 1000$, the cylinder temperature is a function of the axial distance alone, i.e., $T_w = T_w(x)$. This is because, as the value of γ increases, the characteristic Biot number becomes smaller. Thus, the integral solution becomes closer to the finite difference solution as the value of γ increases from 1 to 1000. Note that, for $\gamma = 1$, the centerline temperature remains unchanged until $\xi = 300$, i.e., $\theta_{wcl} = 0$ or $T_w(x,0) = T_{wo}$. The interface temperature θ_i on the other hand, assumes a value that is quite different than θ_{wcl}. This indicates that the conductive thermal wave has not yet penetrated through the entire body of the cylinder, i.e., from the surface to the centerline of the cylinder. For $\xi \geq 300$, the centerline temperature starts to rise, and it eventually approaches the freezing point of the liquid, i.e., $\theta_{wcl} = \theta_i = 1$ or $T_w(x,0) = T_s(x,R) = T_f$ at $\xi = 3.9 \times 10^4$. For $\gamma = 1000$, however, the temperatures behave quite differently. The values of θ_{wcl} and θ_i are almost identical, even at $\xi = 10$, owing to the large thermal conductivity of the cylinder.

The effect of flow-freezing interaction may be assessed by comparing the local convective heat transfer coefficient at the freezing front with the corresponding value for the case of forced convection over a continuous moving cylinder without freezing, as demonstrated in Fig. 3. In this figure, the ratio Nu_x/Nu_{xwo} of local Nusselt numbers for the cases with and without freezing is shown as a function of the normalized dimensionless axial distance $\xi/\xi_{\max}$. If indeed there is no flow-freezing interaction, the ratio Nu_x/Nu_{xwo} would be equal to unity. Thus, any deviation of the value of Nu_x/Nu_{xwo} from unity would serve as a measure of the effect of flow-freezing interaction. As can be seen from the figure, the ratio of Nu_x/Nu_{xwo} is very close to unity, i.e., the effect of flow-freezing interaction is negligible if the Stefan number is very small. At a larger Stefan number, however, the effect of flow-freezing interaction is getting stronger and cannot be neglected. Over the distances where the freeze-coat is growing, i.e., $\xi/\xi_{\max} < 1$, the local Nusselt number at the freezing front is considerably larger than the one predicted by the forced convection without freezing, i.e., Nu_x/Nu_{xwo} is considerably above unity. Physically, the effect of freezing is similar to the effect of wall suction, which increases the local convective heat transfer rate. In this case, using the conventional solution without freezing to describe the local heat transfer at the freezing front would overpredict the freeze-coat thickness. On the other hand, over the axial distances where the freeze-coat is decaying, i.e., $\xi/\xi_{\max} > 1$, the value of Nu_x/Nu_{xwo} is smaller than unity. Physically, the effect of remelting is similar to the effect of blowing at the wall that decreases the local convective heat transfer rate. A comparison between the present results and those for a continuous flat plate[4] indicates that the effect of flow-freezing interaction is less severe in the case of a continuous cylinder. A possible explanation for this is that the flow and heat transfer area is a linearly increasing function of the distance from the surface of a cylinder, whereas it is a constant for a flat plate.

Acknowledgments

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